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Coupled Slots on an Anisotropic Sapphire Substrate

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Abstract—Two analytical approaches are presented for coupled slots on an anisotropic sapphire substrate using the network analytical methods of electromagnetic fields. One is based on the quasi-static approximation and it derives the transformation from the case with the anisotropic substrate to the case with the isotropic substrate. The other is based on the hybrid mode formulation and it gives the dispersion characteristics.

I. INTRODUCTION

PROPAGATION CHARACTERISTICS of coplanar transmission lines have received considerable attention. The slot line [1] has been analyzed using the hybrid mode formulation [2]–[4]. However, the dispersion characteristic of coupled slots has been evaluated based on the first-order approximation [3], [5]. Moreover, only the case with isotropic substrate has been treated.

In this paper, a method of analysis of coupled slots on an isotropic and/or anisotropic substrate is presented. This method contains two approaches: the quasi-static and hy-

brid mode formulation. The quasi-static approach, based on the network analytical methods of electromagnetic fields [6] and variational techniques, shows that coupled slots on an anisotropic sapphire substrate can be transformed into the equivalent coupled slots on an isotropic substrate. The hybrid mode formulation, which is an extension of the treatment in [5], gives the dispersion characteristics of the dominant and first higher order mode, and it suggests the close relation between the first higher order mode of coupled slots and the TM_0 surface wave of a sapphire-coated conductor.

II. ELECTROMAGNETIC FIELD REPRESENTATIONS

The cross section of the coupled slots on a single-crystal sapphire substrate is shown in Fig. 1. The sapphire crystal is uniaxial, but is anisotropic. The permittivity tensor of sapphire is the diagonal matrix as follows:

$$\hat{\epsilon} = \epsilon_0 \begin{bmatrix} \epsilon_{\perp} & 0 & 0 \\ 0 & \epsilon_{\perp} & 0 \\ 0 & 0 & \epsilon_{11} \end{bmatrix}. \quad (1)$$

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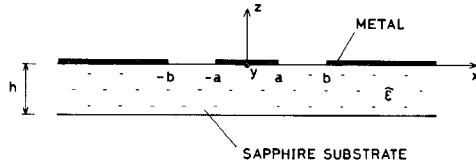


Fig. 1. Coupled slots.

Since the structure in Fig. 1 is symmetric with respect to the $x=0$ plane, we can place either a magnetic (odd modes) or an electric wall (even modes) at this plane of symmetry, and we only need to consider the right half of the structure.

As a first step we express the transverse fields in the regions (1) $z>0$, (2) $0>z>-h$, and (3) $-h>z$ by the following Fourier integral:

$$\begin{Bmatrix} E_t^{(i)} \\ H_t^{(i)} \end{Bmatrix} = \sum_{l=1}^2 \int_{-\infty}^{\infty} e^{-j\beta_0 y} \begin{Bmatrix} V_l^{(i)}(\alpha; z) & f_l(\alpha; x) \\ I_l^{(i)}(\alpha; z) & g_l(\alpha; x) \end{Bmatrix} d\alpha, \quad i=1,2,3 \quad (2)$$

where

$$\begin{aligned} f_1 &= \frac{j}{\sqrt{2\pi K}} K e^{-j\alpha x}, & f_2 &= f_1 \times z_0 \\ g_l &= z_0 \times f_l, & l &= 1, 2 \\ K &= x_0 \alpha + y_0 \beta_0, & K &= |K| \end{aligned} \quad (3)$$

where β_0 is the propagation constant in the y -direction, x_0 , y_0 , and z_0 are the x -, y -, and z -directed unit vectors, respectively, and $l=1$ and $l=2$ represent E waves ($H_z=0$) and H waves ($E_z=0$), respectively. Mode voltages $V_l^{(i)}$ and currents $I_l^{(i)}$ in each region can be related to the mode voltage at the slot surface $z=0$ by applying the continuity condition at $z=-h$:

$$\begin{aligned} V_l^{(i)}(\alpha; z) &= T_l^{(i)}(\alpha; z|0) \bar{V}_l(\alpha) \\ I_l^{(i)}(\alpha; z) &= Y_l^{(i)}(\alpha; z|0) \bar{V}_l(\alpha) \end{aligned} \quad (4)$$

where the modal Green's functions $T_l^{(i)}$ and $Y_l^{(i)}$ are given in the Appendix. The mode voltage at the slot surface \bar{V}_l in turn is expressed in terms of the transverse electric field at the slot surface \bar{E} :

$$\bar{V}_l(\alpha) = \int_{-\infty}^{\infty} f_l^*(\alpha; x') \cdot \bar{E}(x', y') e^{j\beta_0 y'} dx' \quad (5)$$

and \bar{E} may be expressed as

$$\bar{E}(x, y) = \{x_0 e_x(x) + y_0 e_y(x)\} e^{-j\beta_0 y}. \quad (6)$$

The electromagnetic fields in each region can be obtained by substituting (4) into (2).

III. METHOD OF SOLUTION

Two different approaches are presented here: one is based on a quasi-static approximation and it derives the equivalent coupled slots on the isotropic substrate for the odd mode; the other uses the hybrid mode formulation and

it gives the dispersion characteristic of coupled slots for the even and odd mode.

A. Quasi-Static Approximation

In the quasi-static approximation, the parameters of the transmission line can be obtained from the line capacitance per unit length C for the case with substrate and C_0 for the case without substrate.

We will derive a stationary value expression of the line capacitance per unit length of coupled slots. The charge distributions on the conductors at $z=0$ can be obtained by the use of the equation of continuity

$$\sigma(x) = \frac{j}{\omega} \nabla \cdot \mathbf{J}(x) \quad (7)$$

where \mathbf{J} is the current density on the conductor and can be obtained by

$$\mathbf{J}(x) = z_0 \times \{ \mathbf{H}_t^{(1)}(+0) - \mathbf{H}_t^{(2)}(-0) \}. \quad (8)$$

Under the quasi-static approximation, namely $\omega \rightarrow 0$ and $\beta_0 \rightarrow 0$, the charge distribution $\sigma(x)$ can be expressed by using (2), (4), (7), and (8):

$$\sigma(x) = \int_0^\infty \int_a^b F(\alpha) \cos(\alpha x) \sin(\alpha x') e_x(x') dx' d\alpha \quad (9)$$

where

$$F(\alpha) = \frac{2\epsilon_0}{\pi} \left\{ 1 + \frac{1 + \sqrt{\epsilon_{\parallel} \epsilon_{\perp}} \tanh\left(\sqrt{\frac{\epsilon_{\perp}}{\epsilon_{\parallel}}} h \alpha\right)}{1 + \frac{1}{\sqrt{\epsilon_{\parallel} \epsilon_{\perp}}} \tanh\left(\sqrt{\frac{\epsilon_{\perp}}{\epsilon_{\parallel}}} h \alpha\right)} \right\}. \quad (10)$$

The total charge located between $-x$ and x becomes

$$\begin{aligned} Q(x) &= \int_{-x}^x \sigma(x_1) dx_1 \\ &= \int_0^\infty \int_a^b G(\alpha; x|x') e_x(x') dx' d\alpha \end{aligned} \quad (11)$$

where

$$G(\alpha; x|x') = \frac{2F(\alpha)}{\alpha} \sin(\alpha x) \sin(\alpha x'). \quad (12)$$

When x lies within the slot region $a < x < b$, $Q(x)$ is equal to a constant Q , that is, the total charge on the center strip:

$$Q(x) = Q_0 = \int_0^\infty \int_a^b G(\alpha; x|x') e_x(x') dx' d\alpha, \quad a < x < b. \quad (13)$$

Equation (13) is the basic equation for obtaining the stationary value expression of the line capacitance C . We multiply (13) by $e_x(x)$, and integrate over the slot region $a < x < b$. The resultant equation is written as

$$\begin{aligned} Q_0 \int_a^b e_x(x) dx \\ = Q_0 V = \int_a^b \int_a^b \int_0^\infty e_x(x) G(\alpha; x|x') e_x(x') d\alpha dx' dx \end{aligned} \quad (14)$$

where V is the potential difference between the center strip and the ground plane

$$V = \int_a^b e_x(x) dx. \quad (15)$$

Hence, the line capacitance C can be expressed as follows:

$$C = \frac{Q_0}{V} = \frac{\int_a^b \int_a^b \int_0^\infty e_x(x) G(\alpha; x|x') e_x(x') d\alpha dx' dx}{\left\{ \int_a^b e_x(x) dx \right\}^2} \quad (16)$$

which has the property that C is stationary for small variation in $e_x(x)$, and it is always larger than the exact value.

From (10), (12), and (16), the line capacitance of the odd mode of the coupled slots on this specified anisotropic substrate coincides with that of the equivalent coupled slots on the isotropic substrate, of which the effective substrate height and the relative permittivity are $\sqrt{\epsilon_\perp/\epsilon_\parallel} h$ and $\sqrt{\epsilon_\perp\epsilon_\parallel}$, respectively. It can be shown that this result is identical to that derived by Kobayashi *et al.* [7].

B. Determinantal Equation for the Propagation Constant

In this section the method for analyzing the dispersion characteristic of coupled slots is presented. This method is analogous to that for the single slot line on a dielectric substrate [4] but is extended to treat the coupled slots on an anisotropic substrate.

Applying the continuity condition at the slot plane $z=0$ to the integral representation of the magnetic field in Section II, we obtain the integral equation on the slot field and the unknown propagation constant.

The determinantal equation for the propagation constant β_0 can be obtained by applying Galerkin's procedure to the integral equation above. In this procedure, we first expand the unknown slot field $e_x(x)$ and $e_y(x)$ in terms of known basis functions $f_{xk}(x)$ and $f_{yk}(x)$ as follows:

$$\begin{aligned} e_x(x) &= \sum_{k=1}^{N_x} a_{xk} f_{xk}(x) \\ e_y(x) &= \sum_{k=1}^{N_y} a_{yk} f_{yk}(x) \end{aligned} \quad (17)$$

where a_{yk} are unknown coefficients. We will now substitute (17) into the integral equation and apply Galerkin's procedure. After carrying out the integration with respect to x and x' , we obtain the set of simultaneous equations on the unknown a_{yk} . This set of equations is homogeneous, and may be written in the following matrix form:

$$[G(\beta_0)] \begin{bmatrix} \bar{a}_x \\ \bar{a}_y \end{bmatrix} = 0 \quad (18)$$

where $[G(\beta_0)]$ is a square matrix of order $N_x + N_y$, and \bar{a}_x

and \bar{a}_y are column matrices of the unknowns a_{xk} and a_{yk} , respectively.

We finally obtain the determinantal equation for the propagation constant β_0 , by setting the determinant of the coefficient matrix $[G(\beta_0)]$ equal to zero:

$$\det[G(\beta_0)] = 0. \quad (19)$$

IV. NUMERICAL COMPUTATION AND RESULTS

A. Quasi-Static Characteristics

The line capacitance of coupled slots is calculated by applying the Ritz procedure to the variational expression (16). We express the slot field $e_x(x)$ as:

$$\begin{aligned} e_x(x) &= \frac{1}{\sqrt{1 - \left\{ \frac{2(x-S)}{W} \right\}^2}} \\ &+ \sum_{k=1}^N A_k \frac{T_k \left\{ \frac{2(x-S)}{W} \right\}}{\sqrt{1 - \left\{ \frac{2(x-S)}{W} \right\}^2}}, \\ W &= b-a, \quad S = (a+b)/2 \end{aligned} \quad (20)$$

where $T_k(y)$ are Chebyshev's polynomials of the first kind and A_k are variational parameters. Note that the first term in (20) is the slot field of a single slot in free space and the other terms are introduced for the evaluation of the slot field of coupled slots with substrate. The best approximation is obtained by substituting (20) into (16) and requiring

$$\frac{\partial}{\partial A_i} C = 0, \quad i = 1, 2, \dots, N. \quad (21)$$

A special case is considered for the comparison with the rigorous solution

$$\epsilon_\perp = \epsilon_\parallel = 1.$$

In this case the line capacitance C_0 can be obtained analytically by using a conformal mapping. Table I shows the numerical results for $(\pi/4\epsilon_0)C_0$ of coupled slots without substrate. It indicates that retaining only one term, $N=0$, is sufficient for the large spacing a/b , while the higher terms are needed for small values of a/b where the interaction between slots is important.

B. Dispersion Characteristics

The choice of basis functions is important for the numerical computation. We adopt the following families of functions for basis functions:

$$\begin{aligned} f_{xk}(x) &= \frac{T_{k-1} \left\{ \frac{2(x-S)}{W} \right\}}{\sqrt{1 - \left\{ \frac{2(x-S)}{W} \right\}^2}} \\ f_{yk}(x) &= U_k \left\{ \frac{2(x-S)}{W} \right\} \end{aligned} \quad (22)$$

TABLE I
LINE CAPACITANCE PER UNIT LENGTH OF COUPLED SLOTS
WITHOUT SUBSTRATE ($\pi/4\epsilon_0$) C_0

N a/b	0	1	Conformal mapping
0.2	1.7273	1.6547	1.6529
0.4	2.2093	2.1840	2.1838
0.6	2.7646	2.7566	2.7566
0.8	3.5820	3.5804	3.5804

TABLE II
PROPAGATION CONSTANTS FOR THE DOMINANT MODE $\beta_0/\omega\sqrt{\epsilon_0\mu_0}$

a/W	N _x	N _y	Odd-mode	Even-mode
0.25	1	0	2.4847	2.4045
	1	1	2.6039	2.4488
	2	2	2.4817	2.4018
	3	3	2.4789	2.4015
1.00	1	0	2.4737	2.3994
	1	1	2.4805	2.4071
	2	2	2.4744	2.4004
	3	3	2.4742	2.4004
∞ (Single slot)	1	1	2.3461	
	2	2	2.3457	
	3	3	2.3457	

$$\begin{aligned}\epsilon_{\perp} &= 9.4, & \epsilon_{\parallel} &= 11.6 \\ h &= 1 \text{ (mm)}, & W &= 1 \text{ (mm)} \\ f &= 25 \text{ (GHz)}\end{aligned}$$

where $T_k(y)$ and $U_k(y)$ are Chebyshev's polynomials of the first and second kind, respectively. Note that these basis functions account for the edge effect of the slot field.

Table II shows the results for coupled slots and for a single slot, which is a limiting case as the spacing a/W becomes very large, using the different order of approximation. The reasonable convergence is obtained for each spacing by increasing the number of basis functions, but the rate of convergence for small spacing, a/W , is slower than for large spacing; similarly in quasi-static case, because of the interaction between slots. Therefore, the smaller the spacing, the more basis functions needed.

Fig. 2 shows the odd-mode dispersion characteristic of coupled slots on an isotropic substrate. Results are com-

pared with those from [3]. The values of Knorr *et al.* [3] are based on the first-order approximation that neglects the y -directed electric field component $e_y(x)$ and that corresponds to the case $N_x = 1, N_y = 0$ in our method.

Fig. 3 shows the even- and odd-mode dispersion characteristics of coupled slots on a sapphire substrate, where the normalized propagation constants for the dominant mode are presented and compared with the results of the equivalent coupled slots on the isotropic substrate, which was derived in Section III-A. While this equivalent model was based on the quasi-static approximation for the odd dominant mode, it is a good approximation of the case with sapphire substrate for both the odd and even mode in a wide range of frequencies.

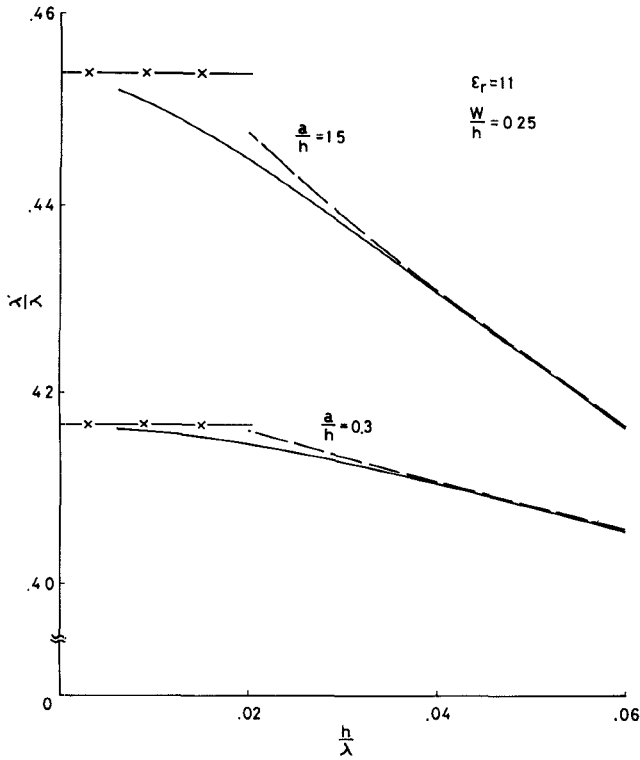


Fig. 2. Normalized guide wavelength. — — — Knorr and Kuchlers' [3]; — × — × — quasi-static; — — — hybrid.

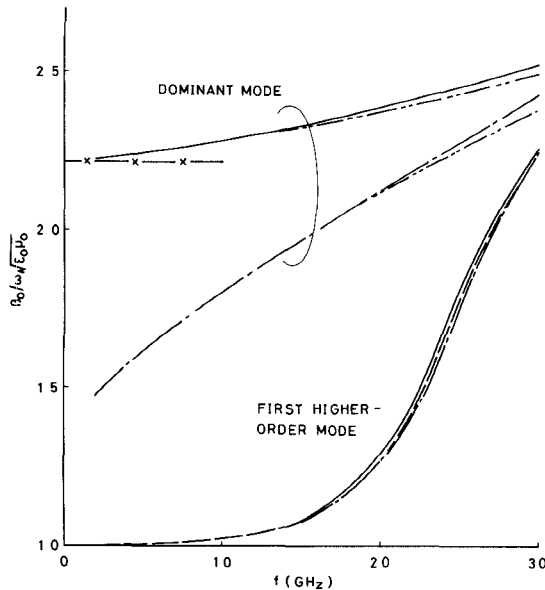


Fig. 3. Normalized propagation constant. $\epsilon_{\perp} = 9.4$, $\epsilon_{\parallel} = 11.6$, $h = 1.0$ (mm), $a = 0.25$ (mm), $b = 1.25$ (mm); — — — odd modes of coupled slots on sapphire; — — — even modes of coupled slots on sapphire; — · — · — dominant modes of equivalent coupled slots; — × — × — quasi-static; — — — TM_0 mode of a sapphire-coated conductor.

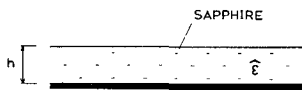


Fig. 4. A sapphire-coated conductor.

Fig. 3 also presents the first higher order mode of coupled slots, compared with the TM_0 , that is, the dominant mode of the sapphire-coated conductor of Fig. 4, and shows a good agreement in a wide frequency band, suggesting the close relation between them. An explanation for this behavior is that, for the TM modes, the slits on the conductor along the propagation direction do not disturb the current flow of these modes.

V. CONCLUSIONS

We presented two analytical approaches for coupled slots using the network analytical methods of electromagnetic fields. One is based on the quasi-static approximation and it derives the transformation from the case with the anisotropic substrate to the case with the effective isotropic substrate. The other, based on the hybrid mode formulation, gives the dispersion characteristics for the dominant and first higher order mode.

Numerical results indicate that the results of the first-order approximation are poor for a narrow spacing and the interaction between slots must be accounted for. Another observation is that the equivalent coupled slots on an isotropic substrate, which is based on the quasi-static approximation, is a good approximation for coupled slots on an anisotropic sapphire substrate over a wide range of frequencies. We have also observed the similarity of the dispersion characteristics between the first higher order mode of coupled slots and the TM_0 mode of a sapphire-coated conductor.

APPENDIX

Modal Green's functions $T_l^{(i)}$ and $Y_l^{(i)}$ are given by

$$T_l^{(1)}(\alpha; z|0) = e^{-\kappa_0 z}$$

$$Y_l^{(1)}(\alpha; z|0) = y_l^{(1)} e^{-\kappa_0 z}$$

$$T_l^{(2)}(\alpha; z|0) = \frac{\cosh\{\kappa_l(z+h)\} + \frac{y_l^{(1)}}{y_l^{(2)}} \sinh\{\kappa_l(z+h)\}}{\cosh(\kappa_l h) + \frac{y_l^{(1)}}{y_l^{(2)}} \sinh(\kappa_l h)}$$

$$Y_l^{(2)}(\alpha; z|0) = -y_l^{(1)} \frac{\cosh\{\kappa_l(z+h)\} + \frac{y_l^{(2)}}{y_l^{(1)}} \sinh\{\kappa_l(z+h)\}}{\cosh(\kappa_l h) + \frac{y_l^{(1)}}{y_l^{(2)}} \sinh(\kappa_l h)}$$

$$T_l^{(3)}(\alpha; z|0) = \frac{e^{\kappa_0(z+h)}}{\cosh(\kappa_l h) + \frac{y_l^{(1)}}{y_l^{(2)}} \sinh(\kappa_l h)}$$

$$Y_l^{(3)}(\alpha; z|0) = -y_l^{(1)} \frac{e^{\kappa_0(z+h)}}{\cosh(\kappa_l h) + \frac{y_l^{(1)}}{y_l^{(2)}} \sinh(\kappa_l h)}$$

where

$$\begin{aligned}\kappa_0 &= \sqrt{K^2 - \omega^2 \epsilon_0 \mu_0} & \kappa_1 &= \sqrt{\frac{\epsilon_{\perp}}{\epsilon_{\parallel}} K^2 - \omega^2 \epsilon_0 \epsilon_{\perp} \mu_0} \\ \kappa_2 &= \sqrt{K^2 - \omega^2 \epsilon_0 \epsilon_{\perp} \mu_0} \\ y_1^{(1)} &= j \frac{\omega \epsilon_0}{\kappa_0} & y_2^{(1)} &= -j \frac{\kappa_0}{\omega \mu_0} \\ y_1^{(2)} &= j \frac{\omega \epsilon_0 \epsilon_{\perp}}{\kappa_1} & y_2^{(2)} &= -j \frac{\kappa_2}{\omega \mu_0}.\end{aligned}$$

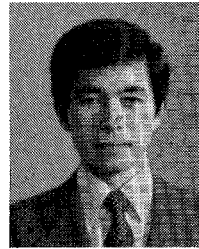
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